Object Discovery via Cohesion Measurement

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Abstract—Color and intensity are two important components in an image. Usually, groups of image pixels, which are similar in color or intensity, are an informative representation for an object. They are therefore particularly suitable for computer vision tasks, such as saliency detection and object proposal generation. However, image pixels, which share a similar real-world color, may be quite different since colors are often distorted by intensity. In this paper, we reinvestigate the affinity matrices originally used in image segmentation methods based on spectral clustering. A new affinity matrix, which is robust to color distortions, is formulated for object discovery. Moreover, a Cohesion Measurement (CM) for object regions is also derived based on the formulated affinity matrix. Based on the new Cohesion Measurement, a novel object discovery method is proposed to discover objects latent in an image by utilizing the eigenvectors of the affinity matrix. Then we apply the proposed method to both saliency detection and object proposal generation. Experimental results on several evaluation benchmarks demonstrate that the proposed CM based method has achieved promising performance for these two tasks.

Index Terms—Cohesion Measurement, Spectral Clustering, Salient Object Detection, Object Proposal Generation.

I. INTRODUCTION

OBJECT detection is one of fundamental tasks in computer vision. In the literature, this problem is conventionally addressed by applying a classifier on all possible hypotheses (i.e., all object candidate regions in an image). At the beginning stage, the candidate regions are exhaustively generated to cover all possible locations and scales based on the sliding window strategy (e.g., [1], [2], [3]). However, the sliding window strategy is computationally expensive. In order to reduce the number of the candidate regions, image segmentation based methods are adopted, which generate much less number of class-independent object proposal windows [4], [5]. As a consequence, a classifier (i.e., an object detector) only focuses on the proposal windows to identify the objects in an image. This kind of methods includes R-CNN [6], SPP-net [7], etc. With the support of proposal windows based on image segmentation, the task of object detection has been relieved from the heavy burden of exhaustive detection, whereas the performance of those methods becomes sensitive to the accuracy of image segmentation. Unfortunately, perfect image segmentation is hard to achieve in general.

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Aiming to circumvent the ill-posed problem of image segmentation, several other generic object proposal generation methods are proposed. For example, the objectness methods [8], [9] are able to generate category-independent proposal windows efficiently, and they can achieve high recall. However, the major disadvantage of these methods is that they are unable to generate a bounding box for an object as precisely as a segmentation based method. Compared with the methods based on image segmentation and object proposal, saliency detection [10], [11], [12] makes a better trade-off between the computational cost and the accuracy for estimating the bounding boxes of objects. The principle of saliency detection is that it only selects the regions which draw the attention of human beings at their first sight [13], [14]. Similar to object detection methods, saliency detection has been adopted as a pre-processing step in various applications. However, most saliency detection methods only extract salient foreground objects from an image, which hinders themselves from being adopted to detect all the objects in an image. Moreover, finding all the objects in an image is beyond the scope of saliency detection.

In this paper, we reinvestigate spectral clustering for image segmentation, and formulate a novel and effective affinity matrix for object discovery. Then, we apply the eigenvectors of the affinity matrix to the problems of saliency detection1 and object proposal generation. The intuition behind the proposed

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1The proposed method is applied to the task of salient object detection in this paper. For simplicity, we use the term of saliency detection.
method is based on a common observation that an object region usually distinguishes itself from its surroundings mainly due to color and intensity differences. In real-world images, image pixels on the same object region share similar real-world color values. Meanwhile, due to variations in lighting conditions and shadow effects, image pixels of an object region in an image are usually not concentrated in the RGB color space. Instead, they are usually distributed within an elongated ellipse region in the RGB space. Different object regions (with different dominant colors) in an image form different elongated ellipse regions with different orientations. Omer and Werman [15] are the first who observed this case. In their work, the elongated ellipse regions are called as color lines, and this phenomenon is called as the color line phenomenon. The color lines are essentially caused by color distortions inside different object regions. Because of the color lines phenomenon, the image pixels, which share a dominant color, may be quite far away from each other when being measured with the Euclidean distance metric. As a result, a heuristical way to segment different latent object regions in an image is to propose an image segmentation method based on color lines. Then objects contaminated by color distortions can be segmented conveniently based on the obtained object regions. On the other hand, image segmentation methods based on spectral clustering (such as [16], [17]) also face the problem of color distortions. However, the affinity matrices used in those spectral clustering based methods do not consider the influence of color distortions. That is the reason why some image segmentation methods based on spectral clustering do not work well on natural scene images.

To address the above issues, a novel affinity matrix, which is robust to color distortions, is proposed in this study. In addition, a cohesion measurement for object regions is also adopted in the proposed affinity matrix. Generally speaking, different object regions obtain different cohesion measurement values. Thus, the problem of object discovery can be treated as the problem of detecting different local maxima of the cohesion measurement for an image. Different from traditional spectral clustering methods (e.g., [17], [16], [18]), the eigenvalues of the proposed affinity matrix can be used to measure the cohesion of object regions, and the corresponding eigenvectors contain important information of the objects. Fig. 1 outlines the general idea of the proposed method. The original images are shown in Fig. 1(a). The detected salient objects, which correspond to the top-2 ranked eigenvectors of the affinity matrix, are shown in Fig. 1(b)-(c), respectively.

As the major contribution of this paper, a novel method, based on a new Cohesion Measurement (CM), is proposed to discover objects latent in an image. Inspired by the concept of color lines, some existing image segmentation methods based on spectral clustering are reinvestigated and a novel affinity matrix, which is robust to color distortions, is proposed in this study. Since different eigenvectors of the formulated affinity matrix usually correspond to different latent objects in an image, the proposed method can be applied to various computer vision tasks, such as saliency detection, object proposal generation, and so on.

The remainder of this paper is organized as follows. In Section 2, we review some existing spectral clustering methods for image segmentation. In Section 3, we detail the construction procedure of the affinity matrix and the proposed method for object discovery. Moreover, we show the advantages of the proposed method over several existing spectral clustering methods for image segmentation. In Section 4, we apply the proposed method to saliency detection and object proposal generation. In Section 5, we evaluate the proposed method and present experimental results on several challenging benchmark datasets. Finally, we draw conclusions in Section 6.

II. RELATED WORK

An affinity matrix plays an important role in the spectral graph theory [19]. For example, Shi et al. [16] propose to solve the perceptual grouping problem using normalized cuts. They set up a weighted graph and set the weight of each edge connecting two nodes to be a measure of the affinity between the two nodes. Then image segmentation is treated as a graph partitioning problem, which is to find the minimum cut of a graph by solving a generalized eigenvalue problem on the constructed Laplacian matrix. The second-to-last eigenvectors turn out to be indicator vectors for partitioning the graph. Perona et al. [17] consider an asymmetric variant of the cost function in [16], and define one of the two subsets of a graph to be the foreground and define its complement to be the background. Based on the modified normalized cuts, they derive a foreground cut method by performing affinity factorization on the affinity matrix. The first eigenvector of the affinity matrix with the largest eigenvalue contains salient objects.

The application of the proposed CM-based method on saliency detection is similar to the above-mentioned foreground cut method. The first several eigenvectors of the proposed affinity matrix ranked by its eigenvalues in descending order can be used to detect salient objects. However, those methods aforementioned (i.e., [16] and [17]) do not consider color distortions in the affinity matrices. Image pixels in a small patch of an image, sharing the same real-world colors, may be similar to each other, while image pixels across the whole object may be quite different because of color distortions. Therefore, instead of defining an affinity matrix for all pairwise pixels in an image, we define an affinity matrix based on local affinity of image pixels. LPP [20] builds a graph incorporating the neighborhood information of input data and computes an affinity matrix to map the input data points into a subspace. Although LPP computes an affinity matrix based on a small local window, it does not consider color distortions. Several other spectral clustering methods (e.g., [21], [22]) do not consider color distortions as well. Levin et al. [23] formulate an affinity matrix based on color lines for the task of natural image matting, where they use the affinity matrix to construct the Laplacian matrix. The eigenvector corresponding to the smallest eigenvalue of the Laplacian matrix contains a high-quality matte for the task of natural image matting. In this paper, we define a similar affinity matrix based on color lines and explore the eigenvectors of the affinity matrix (instead of the eigenvectors of the Laplacian matrix) for
the task of object discovery. Moreover, we also analyze the theoretical superiority of the proposed method over several existing spectral clustering based methods for object discovery.

III. OBJECT DISCOVERY

In the proposed object discovery method, we formulate a novel affinity matrix, which is robust to color distortions. The affinity values among image pixels can be either positive or negative. Usually, positive affinity values are obtained if the pixels are similar and spatially adjacent. Otherwise, non-positive affinity values are obtained. Then, image pixels are clustered by using the built affinity matrix. During the process of clustering image pixels, a new measure for evaluating the goodness of clusters, which we call as cohesion measurement, is derived. The cohesion value of a cluster becomes large when the number of similar and spatially adjacent pixels grows. While the cohesion value decreases when the pixels of a cluster have more different values. Such differences are often caused by strong edges or the contrast of a salient object over background. Thus, an object, which has image pixels with similar real-world color and compact spatial positions, often obtains a high cohesion value. In this section, the details of the proposed method are given on how to obtain the proposed affinity matrix, which is encoded in the affinity matrix, is used for the task of object discovery.

A. The Affinity Matrix Based on Color Distortions

Due to color distortions, image pixels on an object may share similar color within its small neighboring region. However, image pixels could be quite different from each other across the whole object. To address this issue, we propose an affinity matrix based on local affinity of image pixels. The proposed affinity matrix is inspired by the matting Laplacian matrix defined in [23]. Given a pixel \( p_i(x, y) \) in an image of \( w \times h \), where \( w \) and \( h \) are respectively the width and height of the image, according to [23], the entry of the matting Laplacian matrix \( L \) at \( (i, j) \) is written as:

\[
\delta_{ij} - \frac{1 + (p_i - \mu_k)^T (\Sigma_k + \frac{\tau E}{|\Omega_k|})^{-1} (p_j - \mu_k)}{|\Omega_k|},
\]

where \( k = x \cdot w + y \), and it denotes the index of each window \( \Omega_k \) surrounding the pixel at \( (x, y) \); \( |\Omega_k| \) denotes the number of pixels in the window; \( \mu_k \) and \( \Sigma_k \) are the mean vector and covariance matrix of the pixel values in the window \( \Omega_k \), respectively; \( \delta_{ij} \) is the Kronecker delta. In order to avoid division by zero, a small constant value \( \tau \) and an identity matrix \( E \) are introduced to Eqn (1).

An affinity matrix \( A^M \) can be derived from the matting Laplacian matrix by rewriting \( A^M = D^M - L \), where \( D^M \) is a diagonal matrix with each diagonal element \( D^M_{ii} = \sum_j A^M_{ij} \). The element of the affinity matrix \( A^M \) at \( (i, j) \) is given by Eqn. (2).

\[
A^M_{ij} = \sum_{k((i,j)\in\Omega_k}} \frac{1 + (p_i - \mu_k)^T (\Sigma_k + \frac{\tau E}{|\Omega_k|})^{-1} (p_j - \mu_k)}{|\Omega_k|},
\]

Such affinity matrix is robust to color distortions. For simplification, we discard the scale factor \( \frac{1}{|\Omega_k|} \) (which is a constant for a given window) and rewrite the affinity matrix as Eqn. (3).

\[
A_{ij} = \frac{\sum_{k((i,j)\in\Omega_k}} 1 + (p_i - \mu_k)^T (\Sigma_k + \tau E)^{-1} (p_j - \mu_k)}{|\Omega_k|},
\]

where \( D_{ii} = \sum_j A_{ij} \) and \( D_{jj} = \sum_i A_{ij} \). \( A_{ij} \) keeps the average affinity between any two pixels in an image and the denominator in Eqn. (3) averages the affinity value. Inspired by the mean filter [24], we expect that the average affinity is robust to noises. The main part of the numerator in Eqn. (3) gives an affinity between pixel \( p_i \) and \( p_j \), which is weighted by color component. For convenience, we define the main part of the numerator in Eqn. (3) as Eqn. (4).

\[
A_E = (p_i - \mu_k)^T (\Sigma_k + \tau E)^{-1} (p_j - \mu_k).
\]

\( A_E \) is robust to variations in lighting and shadows, which will be explained in detail next. Substituting Eqn. (4) to Eqn. (3), we have

\[
A_{ij} = \frac{\sum_{k((i,j)\in\Omega_k}} 1 + A_E}{\sqrt{D_{ii} D_{jj}}},
\]

Let \( U \) be the matrix where each column is the eigenvector of the covariance matrix \( \Sigma_k \), and let \( \phi_z \) be the \( z \)th eigenvalue of the covariance matrix \( \Sigma_k \). Then,

\[
A_E = \theta \cdot (U(p_i - \mu_k))^T (U(p_j - \mu_k)),
\]

where \( \theta = \frac{1}{\phi_z + \tau} \) \( (\theta > 0) \) is the \( z \)th element of the weight vector \( \theta \). The derivation of Eqn. (6) can be seen in Appendix A. \( (U(p_i - \mu_k))^T (U(p_j - \mu_k)) \) is the inner product between the vector \( (p_i - \mu_k) \) and the vector \( (p_j - \mu_k) \) in the reference frame whose basis vectors are the columns of the matrix \( U \). Since an orthogonal transformation and a vector translation do not change the inner product, \( (U(p_i - \mu_k))^T (U(p_j - \mu_k)) \) is also the inner product of the image pixels \( p_i \) and \( p_j \), which defines the affinity between \( p_i \) and \( p_j \). In addition, the \( z \)th element \( \theta_z \) of the weight vector \( \theta \) is nearly inversely proportional to the eigenvalue \( \phi_z \). The weight vector \( \theta \) alleviates the problem that a certain color component with a large variance dominates an affinity value, by which the obtained affinity value is robust to color distortions. For example, the saturations of a certain color component in image pixels in a small patch may be quite different because of color distortions. The resulting affinity values obtained by using the inner product based on these image pixels may be quite small but they have essentially a similar color. Fortunately, the certain color component \( z \) with a large variance (i.e., a principle component) corresponds to a large eigenvalue \( \phi_z \) and the corresponding weight value is small. Thus, \( A_E \) is robust to color distortions for representing similarities between image pixels. \( A_{ij} \), which is the mean value of \((A_E + 1)\) over a small window (see Eqn. (5)), is also robust to color distortions. \( A_{ij} \) maintains the average affinity between any two pixels over multiple windows in an image. Usually, positive affinity values are obtained if image pixels in a window share a similar real-world color. Otherwise, non-positive affinity values are obtained.
optimization problem instead of setting a constant value to this correlation, we use the kernel-target alignment function \( A \) between the affinity matrix \( \mathbf{A} \). Assume that we have an oracle \( \tilde{M} \). As a matter of fact, the optimal problem. Typically, \( \Delta \) pixels. Eqn. (7) converts a clustering problem into a labeling maps corresponding to the first six eigenvectors ranked by their eigenvalues in descending order.

**B. Clustering Using the Affinity Matrix**

As shown in Eqn. (3), the affinity matrix \( \mathbf{A} \) is symmetric and it is a local representation for each pixel \( p_i \) within a small window \( \Omega_k \) surrounding \( p_i \). Thus, the matrix \( \mathbf{A} \) is very sparse. The affinity matrix \( \mathbf{A} \) keeps the pairwise affinity between pixels, which is robust to color distortions caused by variations in lighting and shadows. Thus, we can cluster the pixels, which are spatially adjacent and share similar real-world colors, by using the affinity matrix \( \mathbf{A} \). The aim of clustering is to verify whether pairwise pixels in a small window \( \Omega_k \) belong to the same cluster or not, which can be written in Eqn. (7).

\[
M_{ij} = \begin{cases} 
1 & \text{if } A_{ij} > \Delta, k [(i,j) \in \Omega_k] \\
-1 & \text{Otherwise}
\end{cases},
\]

where \( \Delta \) is a threshold, which is used to partition adjacent pixels. Eqn. (7) converts a clustering problem into a labeling problem. Typically, \( \Delta \) is set to a constant value to obtain \( \hat{M} \). As a matter of fact, the optimal \( M \) can be found by solving an optimization problem instead of setting a constant value to \( \Delta \). Assume that we have an oracle \( \tilde{M} \), a measure of quality can be obtained by measuring the Pearson correlation coefficient between the affinity matrix \( \mathbf{A} \) and the oracle \( \tilde{M} \). To measure this correlation, we use the kernel-target alignment function proposed in [25]:

\[
f(K_1, K_2) = \frac{\langle K_1, K_2 \rangle_F}{\sqrt{\langle K_1, K_1 \rangle_F \langle K_2, K_2 \rangle_F}},
\]

where \( K_d \) (d=1 or 2) is the \( d \)th kernel matrix for the image pixels, \( \langle K_1, K_2 \rangle_F = \sum_{j=1}^{N} K_1(p_i, p_j) K_2(p_i, p_j) \) and \( F \) denotes the Frobenius norm. The kernel-target alignment can also be viewed as the Cosine of the angle between two bi-dimensional vectors \( K_1 \) and \( K_2 \). In this paper, \( N \) denotes the number of image pixels. Estimating an optimized oracle \( \tilde{M} \) is equivalent to maximizing the alignment between \( \mathbf{A} \) and the oracle \( \tilde{M} \), which is modelled as:

\[
\hat{M} = \arg \max \ f(\mathbf{A}, \hat{M}) = \arg \max_{\hat{M}} \frac{\langle A, \hat{M} \rangle_F}{\sqrt{\langle A, A \rangle_F \langle \hat{M}, \hat{M} \rangle_F}}.
\]

The kernel-target alignment between \( \mathbf{A} \) and the oracle \( \hat{M} \) measures how the pixels of the same cluster are close to each other, and it also measures how the pixels of different clusters are far away from each other at the same time. If we consider \( \hat{M} = \ell \ell^T \), where \( \ell \) is a \( N \) dimensional vector whose elements belong to \( \{-1, +1\} \), then

\[
f(\mathbf{A}, \ell \ell^T) = \frac{\langle A, \ell \ell^T \rangle_F}{\sqrt{\langle A, A \rangle_F \langle \ell \ell^T, \ell \ell^T \rangle_F}} = \frac{\langle A, \ell \ell^T \rangle_F}{\langle A \rangle_F \langle \ell \ell^T \rangle_F}.
\]

Since \( \mathbf{A} \) is a constant Hermitian matrix once an image is given, multiplying \( f(\mathbf{A}, \ell \ell^T) \) by a constant \( \langle A \rangle_F \) will not change the solution of maximizing the alignment \( f(\mathbf{A}, \ell \ell^T) \). Therefore, equivalently, the solution of Eqn. (9) can be obtained by solving Eqn. (11).

\[
\ell = \arg \max_{\ell \neq 0} f(\mathbf{A}, \ell \ell^T) = \arg \max_{\ell \neq 0} \frac{\ell^T A \ell}{\ell^T \ell}.
\]

According to the Rayleigh-Ritz theorem [26], the optimal solution of Eqn. (11) can be approximated by the first eigenvector of the affinity matrix \( \mathbf{A} \), which is written as Eqn. (12), Eqn. (13) and Eqn. (14).

\[
\lambda_1 \ell^T \ell \geq \ell^T A \ell \geq \lambda_N \ell^T \ell,
\]

where \( \lambda_i \) is the \( i \)th eigenvalue of the affinity matrix \( \mathbf{A} \).

\[
\lambda_{\max} = \lambda_1 = \max_{\ell \neq 0} \frac{\ell^T A \ell}{\ell^T \ell} = \max_{\ell \neq 1} \ell^T A \ell.
\]

\[
\lambda_{\text{min}} = \lambda_N = \min_{\ell \neq 0} \frac{\ell^T A \ell}{\ell^T \ell} = \min_{\ell \neq 1} \ell^T A \ell.
\]

The theorem also shows that the maximum of the alignment \( f(\mathbf{A}, \ell \ell^T) \) is equal to the largest eigenvalue of the affinity matrix \( \mathbf{A} \). The bigger the eigenvalue is, the bigger the alignment value is. The eigenvector corresponding to each eigenvalue is the obtained local maximum solution of Eqn. (11), which is a soft continuous solution (i.e., continuous labels for image pixels). As the consequence, the total number of clusters is equal to the number of eigenvectors.

Usually, different clusters (i.e., different object regions) obtained from different eigenvectors correspond to different eigenvalues. A salient object, which has high contrast relative to its background, corresponds to a higher eigenvalue. These cases will be explained in detail in the following subsection.

**C. Cohesion Measurement for Object Regions**

In this paper, the concept of cohesion is adopted to measure the tightness of a cluster. Cluster cohesion is defined as the sum of the weights of all the edges between the nodes
within a cluster. Another term is cluster separation, which is defined as the sum of the weights of the edges between the nodes from different clusters. Usually, cluster cohesion and cluster separation measure intra-class affinity and inter-class affinity, respectively. Different clustering results lead to different cluster cohesion and cluster separation values. In our case, the cohesion value of a cluster is obtained by performing eigenvalue decomposition on the affinity matrix. As a result, the eigenvalue corresponds to the degree of the cohesion of one cluster. The higher the cohesion value is, the tighter the cluster is. Basically, the eigenvalues of the affinity matrix measure the quality of clusters. Moreover, we expect that different clusters correspond to different object regions. Thus, the eigenvalues of the affinity matrix are treated as the degrees of cohesion for object regions. In addition to being capable of identifying object regions in an image, the cohesion measurement can also be used to find salient objects. Salient object regions in an image, which have high contrast relative to the background, correspond to higher cohesion values.

To further show the relationship between the degrees of cohesion and object regions, the elements of the label vector \( \ell \) for image pixels obtained from Eqn. (11) are analyzed in two cases, which are listed in Eqn. (15).

\[
\max_{\ell^T \ell = 1} \sum_{ij} \ell_i A_{ij} \ell_j \Rightarrow \begin{cases} 
\text{sgn}(\ell_i) = \text{sgn}(\ell_j), & A_{ij} \geq 0 \\
\text{sgn}(\ell_i) \neq \text{sgn}(\ell_j), & A_{ij} \leq 0 
\end{cases}, \quad (15)
\]

where \( \text{sgn}(\cdot) \) is the sign function that returns the sign of a variable. As shown in Eqn. (15), in order to maximize the objective function, \( \ell_i \) and \( \ell_j \) should have the same sign when \( A_{ij} \) is positive. Otherwise, \( \ell_i \) and \( \ell_j \) should have different signs when \( A_{ij} \) is negative. In both cases, the maximum of the objective function is obtained when both \( A_{ij} \) and \( \ell_i \ell_j \) are large positive values or small negative values. This in turn implies that \( \ell_i A_{ij} \ell_j \) has a big value. When \( A_{ij} \) and \( \ell_i \ell_j \) are large positive values, image regions are obtained by checking the signs of labels for spatial adjacent image pixels. In the other case (i.e., \( A_{ij} \) and \( \ell_i \ell_j \) are small negative values), the labeling values \( \ell_i \) and \( \ell_j \) should be far away from each other under the constraint of \( \ell^T \ell = 1 \). Usually, negative affinity values are caused by different pixel values, which are due to the factors such as color distortions, color contrast, strong edges, etc. However, the formulated affinity matrix is robust to color distortions, which are caused by variations in lighting and shadows. As a result, the difference between image pixels due to color distortions will not lead to a big cohesion value. Therefore, the big cohesion value is mainly due to a large number of spatially adjacent similar pixels, high contrast relative to background regions and strong edges, which imply a salient object region.

D. Object Discovery Using the Affinity Matrix

As discussed in Section III-A, because the affinity matrix is robust to color distortions, the clusters of pixels, which share a similar real-world color and spatially adjacency, are obtained by calculating the eigenvectors of the affinity matrix. Different clusters correspond to different object regions. In other words, the object regions can be segmented by using eigen-decomposition on the affinity matrix, which is written as

\[
A \nu = \lambda \nu, \quad (16)
\]

where \( \nu \) and \( \lambda \) are the eigenvectors and eigenvalues of \( A \), respectively.

Generally speaking, one resulting eigenvector corresponds to one latent object region in an image, although it may be affected by some noises. One element in an eigenvector corresponds to one pixel in the image. As a result, it becomes convenient to relate the eigenvectors to the object regions in an image. Given that \( \nu_k \) is the \( k \)-th element of an eigenvector \( \nu \), the pixel value at \((x, y)\) in the object map derived from \( \nu_k \) and \( \nu \) is written as

\[
V(x, y) = \frac{\nu_k - \min(\nu)}{\max(\nu) - \min(\nu)} C, \quad (17)
\]

where \( x = \frac{k}{w} \), \( y = \text{mod}(k, w) \) and \( C \) denotes a constant. Fig. 2 shows the object maps derived from the first six eigenvectors corresponding to the six largest eigenvalues obtained by Eqn. (17). As shown in Fig. 2, several salient objects or object regions are found according to the first several eigenvectors. Moreover, the proposed method also enumerates object regions at different scales corresponding to different cohesion values.
To verify that the proposed method is able to find the cohesive clusters for image pixels, no matter whether they undergo color distortions or not, we illustrate an example in Fig. 3. We synthesize two input images, where each input image contains two rectangles (shown in Fig. 3(a1) and (a2)). The pixels in the small rectangle in Fig. 3(a1) undergoes intensity variation while the pixels in the other rectangles have the same pixel values. The resulting eigenvectors obtained by the proposed method coincide with the conclusion. As shown in Fig. 3(b1) and (c1), the eigenvectors obtained from the input image in Fig. 3(a1), which undergoes color distortions caused by intensity variation, show that the proposed method is robust to intensity variation for object discovery. In addition, the small rectangle in Fig. 3(a1), which undergoes color distortions, is obtained in the first eigenvector. This shows that the high differences of the pixels in an object region correspond a high cohesion value. For the input image in Fig. 3(a2), image pixels in the rectangles are almost identical. The rectangle derived from the first eigenvector (see Fig. 3(b2)) corresponds to the one which has the largest area of image pixels. This shows that the proposed method tends to obtain the cluster with the maximum cohesion value.

E. Relation to Spectral Clustering

We note that a similar spectral clustering method was presented in [23], which solves an eigen-decomposition problem on the matting Laplacian matrix. Although the affinity matrix in the proposed method plays a similar role as the matting Laplacian matrix, the resulting eigenvectors obtained by the two different matrices are quite different (see Fig. 4 for an illustration). In Fig. 4(b), with the Laplacian matrix defined in [23], only sub-optimal clustering results are obtained since the eigenvectors of the matting Laplacian matrix are not valid clusters of image pixels in object regions when the image pixels suffer from color distortions. In contrast, as shown in Fig. 4(c), the proposed method obtains better clustering results than the method in [23].

The proposed method tends to obtain different cohesive clusters by solving an eigen-decomposition problem on the formulated affinity matrix. Because the proposed affinity matrix is robust to color distortions, it can be more effectively used to find cohesive clusters for image pixels than the affinity matrices defined in the other spectral clustering methods. For instance, Shi and Malik [16], Andrew et al. [18] and Perona et al. [17] construct affinity matrices only based on the Euclidean distance of image pixels in the RGB space. Among which, [16] and [18] respectively use the eigenvectors of the Laplacian matrix and the normalized Laplacian matrix derived from the affinity matrices to segment images; While [17] tries to find foreground regions by calculating the singular value decomposition of the constructed affinity matrix. In fact, the essence of the method in [17] is also to use the eigenvectors of the affinity matrix for image segmentation. Fig. 5 shows the object maps derived from the first three eigenvectors of the Laplacian matrix and the normalized Laplacian matrix obtained in these three methods. As shown in Fig. 5, the eigenvectors of the affinity matrices and the Laplacian matrix, which do not consider color distortions, contain many noises. Moreover, LLP [20] also faces a similar problem on the task of image segmentation. In contrast, the clustering results obtained by the proposed method contain less noises (see Fig. 4(c)).

IV. APPLICATIONS OF THE PROPOSED METHOD FOR OBJECT DISCOVERY

In this section, we apply the proposed method to saliency detection and object proposal generation, respectively. Because each object map may contain a part of an object, the way of combining several object maps is also investigated in this section. Furthermore, inspired by the saliency filter method [27], a strategy is proposed to eliminate the noisy regions in the obtained object maps.
A. Saliency Detection

Saliency Detection with a Single Eigenvector: Firstly, we apply the obtained object maps to saliency detection, which aims to segment a salient object. For an object map \( V \) generated by Eqn. (17), the pixel intensities of an object may be reversed (i.e., the pixel values of the salient objects are lower than those of the background). To overcome this problem, we adopt a strategy which is given by Eqn. (18).

\[
\hat{V}_t = |V_t - \eta|,
\]  

where \( t \) is the index of the object maps; \( \eta \) denotes the mean value of the pixels in the hull of each object map \( V_t \) whose elements are the outermost pixels of the object map. A simple way to obtain a binary mask of an object is to use a threshold on the pixel intensities of the obtained object map, which has also been adopted in [28], [29]. The thresholding method is based on a clip-level (or a threshold value) to turn a grayscale image (i.e., the object map in our case) into a binary image. Then an image morphology algorithm can be used on the binary image to generate the mask of objects.

Saliency Detection with Multiple Eigenvectors: In the above proposed saliency detection method, a salient object is detected by considering only one eigenvector of the affinity matrix, in which we expect that one cohesive object region corresponds to one object and the object region is exactly determined by one eigenvector. However, in practice, different eigenvectors may correspond to different parts of an object due to the influence of color variations in the object. Thus, we also propose a saliency detection method, which considers the contributions from multiple eigenvectors to saliency detection. The object map \( V_S \) derived from a combination of \( S \) eigenvectors is defined as:

\[
V_S = \sum_{t=1}^{S} \hat{V}_t.
\]  

Once we obtain the combined object map \( V_S \), the thresholding method is then used on \( V_S \) for detecting salient objects. However, the combination of several eigenvectors may generate some noisy regions, which do not belong to salient objects. Next, we will describe a noise elimination strategy to eliminate the noisy regions.

The Noise Elimination Strategy for Saliency Detection: Ideally, the combined object map obtained from the first several eigenvectors contains salient objects. However, some noisy regions may exist in the combined object map. These noisy regions usually have low intensity values and a fragmented distribution. In contrast, the object regions in the combined object map often have a compact spatial distribution and stand out saliently. Therefore, we eliminate the noisy regions using the saliency measure defined in the salient filter method [27].

In [27], an image is first abstracted into perceptually homogeneous elements using the superpixel segmentation method [30]. Each element corresponds to a superpixel obtained by [30]. Then, the element uniqueness (or contrast) and the element distribution of the homogeneous elements are defined to evaluate the saliency values of these elements. In their implementation, the mean color of each element is obtained as the representative color of the element. Usually, the mean colors of the elements obtained from an object region may be quite different from each other because of color distortions. However, the elements obtained from the combined object map are generally homogeneous. Therefore, the values of the element uniqueness and the element distribution of the segmented elements are calculated on the combined object map instead of the origin image in our implementation. Since the combined object map is a gray image, the mean intensity of each element is calculated in this task as the representative intensity of the element. Based on the element uniqueness and the element distribution, a saliency map is obtained by assigning a saliency value to each element obtained from a combined object map. The thresholding method will then be used on the obtained saliency map to segment salient objects. Fig. (6) shows several examples of the resulting saliency maps obtained by using the proposed noise elimination strategy on several object maps. The noise regions in the object maps can be effectively eliminated when they have low intensity values and a fragmented distribution. However, some regions in a salient object may also be eliminated by the proposed strategy if they have similar pixel values to the background (see the last column in Fig. (6)).

B. Object Proposal Generation

In this subsection, we apply the obtained object maps to the task of object proposal generation. In comparison to saliency detection, object proposal generation is more challenging because it is to recall all possible objects in an image. Inspired by the Edge Boxes [31] and gPb-owt-ucm [32] methods, we use the information from edges, which are a primitive feature of an image, to generate object proposals quickly. The Canny edge detector [33], due to its simplicity and efficiency, is used to detect the edges in an object map. In order to obtain all possible object proposals, the Canny edge detector is performed on each chosen object map corresponding to an eigenvector of the affinity matrix. The number of eigenvectors used in the task of object proposal generation is more than that is used in the task of saliency detection. Similar to saliency detection, the combinations of the first several eigenvectors are also used to detect edges. All the circumscribed rectangles of the connected and closed edges are treated as the targeting object proposals.
As described in [8], color contrast and closed boundary are two important objectness cues. Actually, object regions are obtained by using the pixel differences caused by color contrast and strong edges in the proposed method. The obtained edges from object maps are almost the significant edges in original images, while they may be mixed with a few insignificant edges obtained from noisy regions. Therefore, the proposed method actually uses the two cues (i.e., color and edge) in a cascade way for the task of object proposal generation.

Truncated Objectness Measure: Similar to the task of saliency detection, we use saliency measure to filter out noisy regions in the task of object proposal generation. Although saliency detection and objectness have different evaluation criteria, both of them aim to find object regions latent in an image. Salient objects should obtain high objectness measure scores, while background regions should obtain low saliency measure or objectness measure scores. Thus, we propose to use the saliency measure defined in [27] to measure the quality of the generated proposal windows. Perceptually homogeneous elements, which are obtained by using a superpixel segmentation method, are replaced by the proposal windows obtained from the object maps in this task. The saliency value of a proposal window, which does not hit any object region, is usually lower. Thus we call this measure as the truncated objectness measure.

V. EXPERIMENTS

In this section, we evaluate the performance of the proposed method on the tasks of both saliency detection and object proposal generation, and compare the proposed method with several other state-of-the-art methods.

For saliency detection, experiments are conducted on three challenging benchmark datasets: MSRA10K [34], THUR15K [35] and PASCAL-S [36] (we will introduce these datasets with more details in Sec. 5.1). Precision, recall and $F_β$-measure (written as Eqn. (20)) are adopted as the evaluation measures.

$$F_β = \frac{(1 + \beta^2)\text{Precision} \times \text{Recall}}{\beta^2 \times \text{Precision} + \text{Recall}}$$

Following [28], [29], $\beta^2$ is experimentally set to 0.3, which emphasizes the average precision more than the average recall.

For object proposal generation, experiments are conducted on the PASCAL VOC 2007 dataset [37] and the MS COCO dataset [38]. As an evaluation measure, the Intersection over Union (IoU) metric is adopted, which is written as

$$\text{IoU} = \frac{\text{CBB} \cap \text{GBB}}{\text{CBB} \cup \text{GBB}},$$

where CBB and GBB denote a candidate bounding box and a ground truth bounding box, respectively. IoU is set to 0.5, which is used as a threshold to check whether a CBB hits a GBB.

In our implementation, $Ω_k$ is a $3 \times 3$ window. The number of superpixels used in the process of eliminating noisy regions is set to 150, which is the optimum value suggested in [27]. The constant $C$ in Eqn. (17) is set to 255. All images are resized to the fixed width (width=200px) while keeping the original aspect ratio. In the task of object proposal generation, the circumscribed rectangles of the connected and closed edges are obtained by using the build-in function regionprops in Matlab, which measures the properties of image regions. As shown in the previous section, the proposed method obtains a set of object regions by finding the maximal cohesive clusters of image pixels. The resulting eigenvectors may not contain correct clusters if an image contains a large area of image borders. Therefore, all the borders of the test images, which have a large area of image borders, are cropped in the experiments.

A. Saliency Detection

In the first experiment, we mainly study the influence of selecting different eigenvectors on the performance of the proposed method. In order to study the contribution of different
Fig. 8: Visual comparison of saliency maps obtained by the nine competing methods (b to j) and ground truth (k).

Fig. 9: The PR curves obtained by the proposed method and the other eight saliency detection methods. (a) The results obtained by the nine competing methods on the MSRA10K dataset. (b) The results obtained by the nine competing methods on the THUK15K dataset. (c) The results obtained by the nine competing methods on the PASCAL-S dataset.

eigenvectors to the performance of the proposed method, the first six ranked eigenvectors, which correspond to the first six largest eigenvalues, are used to evaluate the performance of the proposed method. In the experiment, the value of the threshold varies from 0 to 255 to generate the precision-recall (PR) curve. Fig. 7(a) shows that the resulting PR curves obtained by the proposed method with one of the first six ranked eigenvectors using different thresholding values. As shown in the figure, among all the six ranked eigenvectors, the contribution of the first eigenvector to saliency detection is the most significant. The performance of the proposed method drops quickly when one uses the other five eigenvectors which correspond to smaller eigenvalues. This indicates the most significant eigenvector usually corresponds to a salient object in an image since the salient foreground object commonly has a high contrast relative to the background.

In addition, we also evaluate the performance of the proposed method by combining the first several eigenvectors ranked by their corresponding eigenvalues. The obtained PR curves are shown in Fig. 7(b). As can be seen, the performance of the proposed method with the combination of the first two eigenvectors (i.e., E1 and E2) outperforms that of the proposed method with the other combinations. Based on the analysis in Section 3.3, an eigenvector corresponding to a higher eigenvalue usually contains salient object regions with a higher probability. Thus, the proposed method using the first two ranked eigenvectors outperforms the proposed method using the other pairwise combinations of the eigenvectors. We also note that the average precision obtained by the proposed method becomes lower when it uses three or more eigenvectors. The reason is that not all the objects obtained from the first several ranked eigenvectors (especially for the lower ranked eigenvectors) are salient objects. Thus, using the first two eigenvectors (i.e., E1 and E2) for salient object detection is a good tradeoff between recall and precision. Moreover, the PR curves obtained by the proposed method using the
first ranked eigenvector and using the first ranked eigenvector with the proposed noisy region elimination strategy as a post-processing step are also shown in Fig. 7(c) as a comparison. The result shows that the latter scheme achieves better average precision.

In the second experiment, the performance of the proposed method using the combined maps corresponding to the first two ranked eigenvectors and the noisy region elimination strategy is evaluated. We compare the proposed method with several other state-of-the-art saliency detection methods on three public challenging datasets: MSRA10K [35], THUR15K [35] and PASCAL-S [36]. MSRA10K contains 10,000 images from the MSRA dataset [34] with pixel-level saliency labeling. THUR15K contains five classes of images. In each class, there are around 3,000 images, all of which are downloaded from Flickr [39]. Only 6,233 binary salient masks of the corresponding images in THUR15K are given. All the 6,233 labeling salient masks and their corresponding images in the THUR15K dataset are used for evaluation. PASCAL-S contains 850 images from the PASCAL VOC 2010 dataset [37] with pixel-level saliency labeling. Comparisons are made between the proposed method and eight competing methods, namely, RBD [40], GR [41], MNP [42], SVO [43], DSR [44], RC [29], GB [45], and SF [27]. We select these methods because they are representative methods for saliency detection. The nine competing methods can be roughly divided into two categories: pixel-based methods and region-based methods. SF, MNP and GB are the pixel-based methods, while RBD, GR, RC, DSR, SVO and the proposed method belong to the region-based methods. Moreover, because some saliency detection methods for eye fixation (such as [46], [47]) usually obtain low average precision on the MSRA10K and THUR15K datasets since they can only locate salient objects in an image instead of segmenting them, they are not used for comparison in this paper. Fig. 8(b)-(j) show the saliency maps produced by the nine competing methods for the input images shown in Fig. 8(a). Fig. 9 (a) shows the PR curves obtained by all the nine competing methods on the MSRA10K dataset. As can be seen from Fig. 8(a), Fig. 9 (a) and Table I, the proposed method achieves the best average precision in comparison to the other eight state-of-the-art methods. Generally, the region-based methods obtain better results than those obtained by the pixel-based methods. In the pixel-based methods, SF obtains the best average precision while it obtains the worst recall. In the region-based methods, the average precision obtained by RBD and DSR is 0.88, which is higher than that obtained by the other three region-based methods (i.e., GR, RC and SVO) but lower than that obtained by the proposed method. SVO obtains the best average recall since it fuses the cues of both objectness and visual saliency to improve the recall of salient objects. However, SVO obtains the worst average precision since the saliency maps obtained by SVO contain too many noises.

The PR curves obtained by all the nine competing methods on the THUR15K dataset are shown in Fig. 9(b). From the figure, we can see that the proposed method outperforms most of the other methods, especially when the recall is less than 0.7. The quantitative comparison results can be seen in Table II. In terms of average precision and average recall, the ranking of the nine competing methods on the THUR15K dataset is similar to that on the MSRA10K dataset. A difference is that the average precision obtained by SF on the THUR15K dataset is ranked worse relative to the other competing methods. The main reason is that many images in the THUR15K dataset suffer from color distortions, and SF is not robust to color distortions.

Table 1. The average precision, average recall and $F_\beta$-measure obtained by the proposed method and the other eight state-of-the-arts methods for saliency detection on the MSRA10K dataset. The top three best results are highlighted by red, green and blue, respectively.

<table>
<thead>
<tr>
<th>Methods</th>
<th>SF</th>
<th>MNP</th>
<th>GB</th>
<th>RBD</th>
<th>GR</th>
<th>SVO</th>
<th>DSR</th>
<th>RC</th>
<th>Ours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Precision</td>
<td>0.86</td>
<td>0.65</td>
<td>0.69</td>
<td>0.88</td>
<td>0.78</td>
<td>0.59</td>
<td>0.88</td>
<td>0.84</td>
<td>0.92</td>
</tr>
<tr>
<td>Average Recall</td>
<td>0.36</td>
<td>0.55</td>
<td>0.49</td>
<td>0.65</td>
<td>0.63</td>
<td>0.76</td>
<td>0.61</td>
<td>0.65</td>
<td>0.46</td>
</tr>
<tr>
<td>$F_\beta$-measure</td>
<td>0.77</td>
<td>0.64</td>
<td>0.67</td>
<td>0.86</td>
<td>0.76</td>
<td>0.60</td>
<td>0.85</td>
<td>0.82</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Table 2. The average precision, average recall and $F_\beta$-measure obtained by the proposed method and the other eight state-of-the-arts methods for saliency detection on the THUR15K dataset. The top three best results are highlighted by red, green and blue, respectively.

<table>
<thead>
<tr>
<th>Methods</th>
<th>SF</th>
<th>MNP</th>
<th>GB</th>
<th>RBD</th>
<th>GR</th>
<th>SVO</th>
<th>DSR</th>
<th>RC</th>
<th>Ours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Precision</td>
<td>0.53</td>
<td>0.44</td>
<td>0.48</td>
<td>0.57</td>
<td>0.50</td>
<td>0.39</td>
<td>0.61</td>
<td>0.57</td>
<td>0.69</td>
</tr>
<tr>
<td>Average Recall</td>
<td>0.30</td>
<td>0.57</td>
<td>0.53</td>
<td>0.56</td>
<td>0.55</td>
<td>0.73</td>
<td>0.55</td>
<td>0.61</td>
<td>0.42</td>
</tr>
<tr>
<td>$F_\beta$-measure</td>
<td>0.50</td>
<td>0.44</td>
<td>0.48</td>
<td>0.57</td>
<td>0.50</td>
<td>0.41</td>
<td>0.60</td>
<td>0.57</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Table 3. The average precision, average recall and $F_\beta$-measure obtained by the proposed method and the other eight state-of-the-arts methods for saliency detection on the PASCAL-S dataset. The top three best results are highlighted by red, green and blue, respectively.

<table>
<thead>
<tr>
<th>Methods</th>
<th>SF</th>
<th>MNP</th>
<th>GB</th>
<th>RBD</th>
<th>GR</th>
<th>SVO</th>
<th>DSR</th>
<th>RC</th>
<th>Ours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Precision</td>
<td>0.67</td>
<td>0.62</td>
<td>0.64</td>
<td>0.74</td>
<td>0.65</td>
<td>0.73</td>
<td>0.76</td>
<td>0.67</td>
<td>0.78</td>
</tr>
<tr>
<td>Average Recall</td>
<td>0.14</td>
<td>0.40</td>
<td>0.44</td>
<td>0.47</td>
<td>0.49</td>
<td>0.47</td>
<td>0.42</td>
<td>0.51</td>
<td>0.35</td>
</tr>
<tr>
<td>$F_\beta$-measure</td>
<td>0.51</td>
<td>0.59</td>
<td>0.62</td>
<td>0.70</td>
<td>0.62</td>
<td>0.69</td>
<td>0.71</td>
<td>0.65</td>
<td>0.72</td>
</tr>
</tbody>
</table>
TABLE IV: The mean of the running time used by the nine competing methods on the MSRA10K dataset. The top three best results are highlighted by red, green and blue, respectively.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Code</th>
<th>Running Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF</td>
<td>MATLAB+C</td>
<td>0.01</td>
</tr>
<tr>
<td>MNP</td>
<td>MATLAB+C</td>
<td>27.12</td>
</tr>
<tr>
<td>GB</td>
<td>MATLAB+C</td>
<td>0.38</td>
</tr>
<tr>
<td>RBD</td>
<td>MATLAB+C</td>
<td>0.36</td>
</tr>
<tr>
<td>GR</td>
<td>MATLAB+C</td>
<td>0.64</td>
</tr>
<tr>
<td>SVO</td>
<td>MATLAB+C</td>
<td>4.32</td>
</tr>
<tr>
<td>DSR</td>
<td>MATLAB+C</td>
<td>57.2</td>
</tr>
<tr>
<td>RC</td>
<td>C</td>
<td>0.14</td>
</tr>
<tr>
<td>Ours</td>
<td>MATLAB</td>
<td>3.21</td>
</tr>
</tbody>
</table>

In terms of running time, the average execution time of the proposed method (implemented in Matlab on a 4 GHz 32GB RAM PC) for saliency detection on each image is about 3.2 seconds. Most running time is used to perform eigendecomposition on the affinity matrix. The running time of all the nine competing methods is listed in Table IV. As can be seen, the top three fast methods are respectively SF, RC and RBD, which are implemented in MATLAB+C or C. Although, the proposed method is implemented in Matlab only, it is more efficient than DSR, MNP and SVO. The running time of proposed method can be further reduced with more efficient implementation in C/C++.

B. Object Proposal Generation

In this section, we evaluate the performance of the proposed method for object proposal generation via the proposed cohesion measurement. The performance of the proposed method is evaluated on the PASCAL VOC 2007 dataset and the MS COCO dataset. The PASCAL VOC 2007 dataset contains 9,963 images in which all the object instances from twenty categories are manually annotated by bounding-boxes. Since there is no training step required in the proposed method, we only report the results obtained by the proposed method on the test dataset of the PASCAL VOC 2007 dataset, which consists of 2,507 images. The MS COCO dataset consists of 91 common object categories and 40,137 images in total. All the 40,137 images are used in the experiments.

The IoU metric is used as the evaluation criterion for object proposal generation. In the implementation, we use the first eighty ranked eigenvectors and all the pairwise combinations of the first six ranked eigenvectors to generate object proposals. If the IoU between two candidate bounding boxes (CBBs) is greater than 0.95, the CBB with a lower objectness score is removed from the pairwise CBBs. Then each of the remaining CBBs is assigned with an objectness score using the truncated objectness measure. All the CBBs, whose objectness scores are zero, are also removed. The resulting ranked CBBs are used for evaluation.

We compare the proposed method with several state-of-the-art methods for object proposal generation, and evaluate the recall obtained by the competing methods when varying the number of object proposals. We select the following five
The proposed method for object proposal generation are shown in Fig. 12. Compared with the other state-of-the-art object proposal generation methods, no training step is required in the proposed method. Only two primitive features (i.e., color and edge) are used to locate the objects in an image. Since the proposed method fuses the two objectness cues, it obtains better recall than that obtained by most of the other most competing methods when using a large number of proposals.

The running time of the competing object proposal generation methods is listed in Table V. The top three fastest methods are BING, EdgeBoxes and the proposed method, respectively. However, the implementation of the proposed method is totally in Matlab without interfacing with C as BING and EdgeBoxes do. Generally, the running time of the edge based methods is lower than that of the image segmentation based methods. The slowest method is CPMC, which requires more than 200 seconds to deal with each image in the Pascal VOC 2007 dataset on average.

VI. CONCLUSION

In this paper, we have proposed a novel method to discover objects latent in an image. We propose to encode cohesion measurement into a new affinity matrix, which is robust to color distortions. Images pixels from the same object region are more effectively clustered by using the formulated affinity matrix compared with the traditional spectral clustering methods. Moreover, the problem of object discovery is solved by utilizing the eigenvectors of the proposed affinity matrix, and the corresponding eigenvalues turn out to be an appropriate cohesion measurement for detecting a object region. In addition, we have also shown that how the proposed method is applied to the tasks of saliency detection and object proposal generation. As shown in the experiments, the proposed method is more effective than most of the other state-of-the-art methods on the two tasks. Moreover, since the proposed method dose not require a training step, it can be directly applied to the other related computer vision tasks, such as image segmentation, object tracking, etc.

APPENDIX A

DERIVATION OF EQN. (6)

Since the covariance matrix $\Sigma_k$ is a symmetric and positive semidefinite matrix, the covariance matrix $\Sigma_k$ can be decomposed by using the SVD decomposition method as follow:

$$\Sigma_k = U \Lambda U^T,$$

where $U$ is an orthogonal matrix whose each column is the eigenvector of the covariance matrix $\Sigma_k$ and $\Lambda$ is a diagonal matrix with non-negative diagonal entries $\phi_z$. $\phi_z$ is the $z$th eigenvalue of the covariance matrix $\Sigma_k$. Then

$$\left(\Sigma_k + \tau E\right)^{-1} = \left(U \Lambda U^T + \tau E\right)^{-1} = \left(U(\Lambda + \tau E)U^T\right)^{-1},$$

where $E$ is an identity matrix. Using the fact that $U^{-1} = U^T$, we have,

$$\left(U(\Lambda + \tau E)U^T\right)^{-1} = U^T(\Lambda + \tau E)^{-1}U.$$
Fig. 12: Qualitative examples showing the results obtained by the proposed method. The GBBs are shown in the green solid lines and the correct CBBs with IoU higher than 0.5 are shown in the dash lines with different colors.

Let $\tilde{p}_i = (p_i - \mu_k)$ and $\tilde{p}_j = (p_j - \mu_k)$ denote the new representations of pixels $p_i$ and $p_j$, respectively. Then, $A_E$ can be transformed into

$$A_E = \tilde{p}_i^T U^T (\Lambda + \tau E)^{-1} U \tilde{p}_i$$

$$= (U \tilde{p}_i)^T (\Lambda + \tau E)^{-1} (U \tilde{p}_i)$$

$$= \theta \cdot (U(p_i - \mu_k))^T (U(p_j - \mu_k))$$

where $0 < \theta = \frac{1}{\phi \tau + \tau}$. 

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China under Grants U1605252, 61472334, 61572408 and 61571379, and by the Key Research Program of the Chinese Academy of Sciences under Grant KGZD-EW-T03. We would like to thank the editor and anonymous reviewers for their great efforts in improving the quality of this paper.

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